Recirculation in Swirling Flow: A Manifestation of Vortex Breakdown

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The addition of a sufficiently high degree of swirl to flow into a circular pipe produces a limited region of reversed flow. Such a vortex breakdown, as it is termed, represents a zone of transition from a supercritical to a subcritical flow state. If the flow remains subcritical, an unavoidable consequence is that the geometry and conditions downstream directly affect the upstream flow up to, and including, the breakdown region. LDA measurements of the swirl and axial velocity components, as well as the corresponding streamline patterns, are presented for water flow in a model based upon the geometry of a swirl combustor. It is shown that a strong exit contraction (55% of the diameter) has practically no influence on a flow which reverts to supercritical, whereas even a weak contraction (15% of the diameter) has a significant influence on a flow which remains subcritical. It is argued that a cold flow is likely to be totally unrepresentative of a reacting flow through the same geometry, and, also, that great care has to be taken over the boundary conditions to be imposed for the numerical computation of subcritical flows.

I. Introduction

It has long been recognized[1] that local flow reversal can be produced in a circular duct flow by introducing a sufficiently high degree of swirl into the flow. Such a swirl-induced zone of recirculation is the basis for one of the classical techniques for flame stabilization in combustion chambers and furnaces. Considerable effort has been expended to understand the mechanism which produces the flow reversal and the conditions under which it occurs, and it is now generally accepted that (for nonreacting flows, at least) the phenomenon represents a transition—termed vortex breakdown—from supercritical to subcritical flow, somewhat akin to a shock wave in compressible flow or a hydraulic jump in open-channel flow. In this context, the term critical refers to a flow for which long inertia waves (i.e., those with the highest phase speed) are stationary with respect to the flow. If the flow is supercritical, such waves are swept downstream from the originating disturbance, while for subcritical flow these waves propagate upstream against the flow, carrying with them information about the nature of the downstream geometry and conditions. If the axial velocity is uniform, then a convenient rule of thumb, first established by Squire,[2] is that a flow is subcritical if the maximum swirl velocity exceeds the axial velocity. More generally, it is necessary to make this determination using Benjamin's[3] critical equation.

Gore and Ranz,[4] Syred and Beer,[5] Lilley,[6] and Vu and Gouldin,[7] among others, have suggested that the flow recirculation produced in swirl-stabilized combustion systems is related to vortex breakdown without, however, recognizing the far-reaching implications of this association of ideas. The subcritical nature of the downstream flow, for example, is directly responsible for the influence on the recirculation zone of a contraction far downstream. It is perhaps worth remarking that a flow without swirl may be regarded as infinitely supercritical. In such a case (for an incompressible flow) the influence of an exit contraction would be twofold: first, a global increase in pressure upstream of the exit (for a given flow rate); and, second, an acceleration of the flow within the immediate vicinity of the exit. These two effects are also present for swirling flow, whether supercritical or subcritical. In the latter case, however, a further complicating influence extends throughout the entire subcritical-flow regime, as is discussed in the present paper. Such "unexpected" behavior was reported, for example, in the recent paper by Altgeld et al.[8] for isothermal flow in a model swirl combustor. Though the connection with vortex breakdown was not made by the authors themselves, the transition from supercritical to subcritical conditions is clearly evident in their measurements and observations.

It is evident that the characteristics of swirling flows differ fundamentally from those of nonswirling flows, and that these differences have not been widely recognized. For this reason, a brief discussion is included of those aspects which are particularly relevant here, emphasis being placed on the analogy which can be drawn with compressible flow.

The view that the characteristics of swirl-induced recirculation can only be interpreted from the standpoint of vortex breakdown is supported by experiments carried out under nonreacting conditions (water flow) in a geometry modeled on a swirl-stabilized combustion chamber.

II. Experimental Arrangement

A schematic drawing of the flow geometry used for the present experiments is shown in Fig. 1. Water is supplied from a constant-head tank to a distributor and, hence, to the header tank. An azimuthal component of velocity is imparted to the flow by an axisymmetric array of 32 guidevanes with $m = 36$ mm and chord 50 mm arranged on a pitch circle of radius $r_v = 120$ mm. Visual observations were made in a tube of diameter $D = 60$ mm and $h = 30$ mm and detailed measurements with $D = 55$ mm and $h = 28.5$ mm. The tube length $L = 350$ mm downstream of an annular segment with inner and outer diameters $D_L = 20$ mm and $D_0 = 40$ mm, respectively. For the velocity measurements, a nozzle element of inner diameter $D_N = 32$ mm was also installed as shown in Fig. 1. In a simple swirl combustor, the centerbody (inner cylinder) would correspond to the fuel-injection lance and the outer cylinder/nozzle combination to the stabilizing element for the downstream recirculation region. The three variable parameters were the guidevane angle ϕ, the exit-contraction diameter D_E, and the volumetric flow rate Q.

Received Aug. 9, 1983; revision submitted Feb. 24, 1984.

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Measurements of the mean axial and swirl velocity components were made in the cylindrical tube ($D=55$ mm), at a series of axial locations at 5-mm intervals up to 80 mm from the end of the centerbody, as well as at 100 mm. A further pair of profiles was measured at 215 mm to obtain the "asymptotic" behavior. The measurements were made with a prototype of the BBC-Goerz dual-beam backscatter laser Doppler anemometer (LDA). This instrument employs a 5 mW He-Ne laser with a Bragg cell after the beam splitter to shift the frequency of one beam by 40 MHz, thereby permitting the determination of both the magnitude and the direction of the velocity component being measured. The Doppler-signal quality was improved by seeding the flow with 5.7-μm-diam latex particles. At each axial location, measurements were made at 1-mm traverse intervals (corresponding to about 1.3-mm intervals within the flow) across a diameter of the tube. Averages were formed using 100 samples at each location, the criterion due to Chauvenet[9] being used to reject spurious data.

Flow visualization was carried out in the 60-mm-diam tube using a fluorescent dye (fluorescein) injected radially into the flow through a narrow slit 0.5 mm from the end of the centerbody. The flow was illuminated by sweeping the beam of an argon-ion laser back and forth in a diametral plane at a frequency of about 4 kHz.

III. Influence of Swirl on the Flowfield

We consider first the succession of events which are observed when the swirl is increased from zero and both the flow rate and the geometry of the flow device (i.e., downstream of the guide vanes) remain unchanged. For convenience we introduce a swirl number β which is defined to be the ratio of the azimuthal and axial velocity components at the surface of the central cylinder within the annular cylindric section. For a potential flow we then have

$$\beta=\frac{\Gamma(D_0^2-D_L^2)}{4QD_L}=\frac{(D_0^2-D_L^2)r_v\sin\phi}{4D_L h(r_v\cos\phi-m)} \tag{1}$$

where Γ is the circulation and Q the volumetric flow rate. For the experiments discussed in this section the tube diameter $D=60$ mm, the flow rate $Q=0.073$ ℓ/s, corresponding to a tube Reynolds number $4Q/\pi D\nu=1352$ (mean water temperature 14.7°C), and neither the nozzle element nor the exit-contraction element were installed. As will be seen, the velocity measurements were performed at a considerably higher Reynolds number (7000) in order to ensure an acceptably high rate of data acquisition using the LDA system. Flow visualization is more conveniently carried out at a lower Reynolds number to reduce the rate at which dye is dispersed.

At zero swirl ($\beta=0$) the recirculation zones downstream of the centerbody and the step increase in the tube diameter exhibit the usual wake-like character. When the swirl is increased, the axial-radial flowfield does not change significantly until a certain critical swirl number β_E is reached at which an isolated axisymmetric recirculation zone appears in the downstream section of the tube. It is this recirculation zone which we (and others) identify with vortex breakdown. As the swirl is increased further the recirculation zone moves upstream in the vortex tube until at a swirl number β_0 the wake downstream of the central cylinder and the isolated recirculation zone join together. The flow-visualization photograph in Fig. 2 shows the wake behind the central cylinder and the vortex-breakdown structure for a swirl number $\beta=2.28$ ($\phi=60$ deg) in the interval $\beta_E<\beta<\beta_0$.

The prevalent view[10,11] is that there is a fundamental difference between a breakdown having the spiral appearance seen here and the axisymmetric or bubble form. The present authors[12] have disputed this idea, believing rather that vortex breakdown is an essentially axisymmetric phenomenon and that the spiral structure is a result of instability and rollup of a layer of rotational fluid separating the outer irrotational fluid from an inner bubble of practically stagnant fluid. Measurements of the mean flow structure by Beer and Chigier[13] and Rhode et al.,[14] as well as those reported here, support our view. Further evidence is provided by the observations of Sarpkaya[15] and Escudier and Zehnder[16] that both forms of breakdown can appear for the same flow condition, even appearing interchangeably under some circumstances. It would also seem that the precessing vortex core identified by Syred and Beer[5] and others working on problems of swirl combustion represents the same shear-layer instability and rollup.

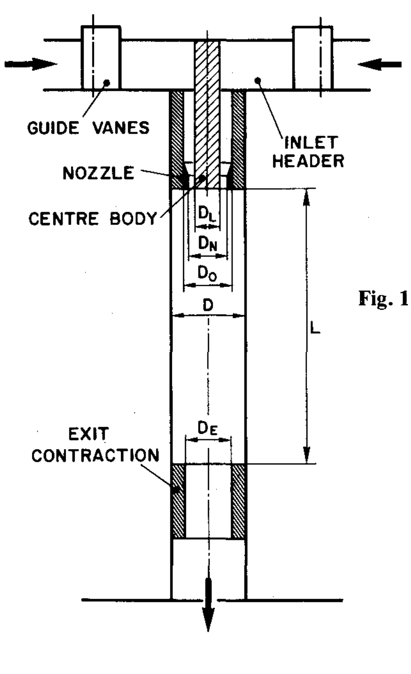

Fig. 1 Vortex-flow apparatus.

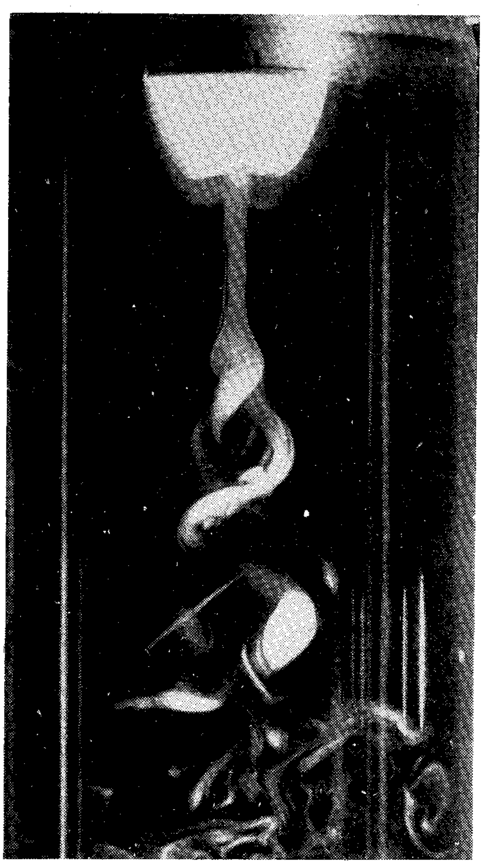

Fig. 2 Flow visualization for separate wake and breakdown recirculation zones: $\phi=60$ deg, $\beta=2.28$, $Re=1352$.

The fact that $\beta_0 \neq \beta_E$ is due to viscous effects acting on the vortex core, which cause a slow decrease of β in the downstream direction, although the cross section of the tube does not change. In fact, for moderate and high Reynolds numbers the values β_0 and β_E are only slightly different. When the swirl number exceeds the value β_0 the vortex-breakdown structure together with the wake behind the central cylinder form a single recirculation zone which grows in diameter with increasing β. Such a situation is shown in Fig. 3 for $\beta = 11.8$ ($\phi = 70$ deg). The highly turbulent nature of the flow even at the relatively low Reynolds number (1352) of this observation is fully evident. When the swirl number reaches a certain value β_I, the recirculation zone jumps back into the annular section of the flow device producing an annular vortex-breakdown structure. When β is increased still further, the annular breakdown moves upstream and finally "disappears" at the upstream end wall. In this case (for sufficiently high Reynolds numbers) the vortex flow is subcritical throughout the tube.

We see that the succession of events is rather similar to that of a shock wave moving upstream against a gas flow in a tube (whose cross-sectional area does not decrease in the downstream direction) as the static pressure at some downstream location is slowly increased. Just as the vortex-breakdown structure is stabilized downstream of the annular part of the vortex tube and produces a recirculation zone within a range of swirl numbers $\beta_0 < \beta < \beta_I$, so a shock wave is stabilized, either in a diffuser or after a sudden enlargement of the tube cross section, within a certain range of Mach numbers. It is also the case that a vortex breakdown can be stabilized in a diffuser or downstream of a sudden enlargement of the cross section of a simple tube. Viscosity also plays a rather similar role for quasi-one-dimensional gasdynamics and quasicylindric vortex flows. In the case of subsonic gas flow in a tube of constant cross section, in the absence of any energy exchange through the tube walls, the effect of viscosity is to increase the Mach number in the flow direction. The gas flow remains subsonic to the end of the tube and becomes critical at the end cross section provided the downstream static pressure is chosen to be sufficiently low. Similarly, a subcritical vortex flow in a tube of constant cross section approaches the critical flow state in the downstream direction and, eventually (in a sufficiently long tube), goes through the critical state and becomes increasingly supercritical, mainly under the influence of viscous vortex-core thickening. Here, we observe an interesting difference: in contrast to a subcritical vortex flow, a subsonic gas flow cannot cross the critical flow state due to the influence of viscosity.

A further question of interest concerns the existence of a continuous transition from a subcritical to a supercritical vortex flow which corresponds to the transition from subsonic to supersonic flow in a Laval nozzle. As first shown by Benjamin,[3] changes of vortex core diameter and vortex tube diameter are of the same sign for supercritical flows and of opposite sign for subcritical flows. Furthermore, increasing the vortex tube diameter always brings the flow condition closer to critical whilst decreasing the tube diameter leads away from the critical state. Hence, as proposed by Benjamin,[17] the device for a vortex flow, which corresponds to a Laval nozzle for a gas flow, consists of a gradual enlargement of the tube cross section, suitably chosen to avoid flow separation at the tube wall, followed by a contraction.

To make our ideas about the behavior of the flow in the neighborhood of the critical state more concrete, we assume that the critical state can be represented as a Rankine vortex, i.e.,

$$u(r) = U_c \qquad 0 \le r \le b_c \tag{2}$$

$$v(r) = \omega r \qquad \text{if } 0 \le r < a_c$$
$$= \omega a_c^2 / r \qquad \text{if } a_c \le r < b_c \tag{3}$$

Fig. 3 Flow visualization for combined wake and recirculation zones: $\phi = 70$ deg, $\beta = 11.8$, $Re = 1352$.

where u and v are the axial and azimuthal velocity components, ω denotes the angular velocity within the core, U_c is the constant axial velocity, a and b denote the core and tube radii, and the subscript c refers to critical values. Away from the critical state, the axial velocity component U in the outer potential flow is related to the local vortex core radius a by (see, e.g., Batchelor[18])

$$\frac{U}{U_c} = 1 + \left(\frac{a_c^2}{a^2} - 1\right) \frac{\frac{1}{2} k a J_0(ka)}{J_1(ka)} \tag{4}$$

where

$$k = 2\omega / U_c \tag{5}$$

and J_0 and J_1 denote Bessel functions of the first kind. Eliminating U with the help of the continuity equation for the potential-flow domain,

$$(b^2 - a^2)\, U = (b_c^2 - a_c^2)\, U_c \tag{6}$$

and making use of the substitutions

$$x = ka, \quad y = kb, \quad R = a_c / b_c \tag{7}$$

we obtain

$$\left(\frac{y}{y_c}\right)^2 = \left(\frac{x}{x_c}\right)^2 + \left(\frac{1}{R^2} - 1\right) \Big/ \left\{ 1 + \left[\left(\frac{x_c}{x}\right)^2 - 1\right] \frac{x J_0(x)}{2 J_1(x)} \right\} \tag{8}$$

Making use of the condition for critical flow for a Rankine vortex (see, e.g., Benjamin[3]),

$$\frac{x_c J_0(x_c)}{2 J_1(x_c)} = \frac{-R^2}{1 - R^2} \tag{9}$$

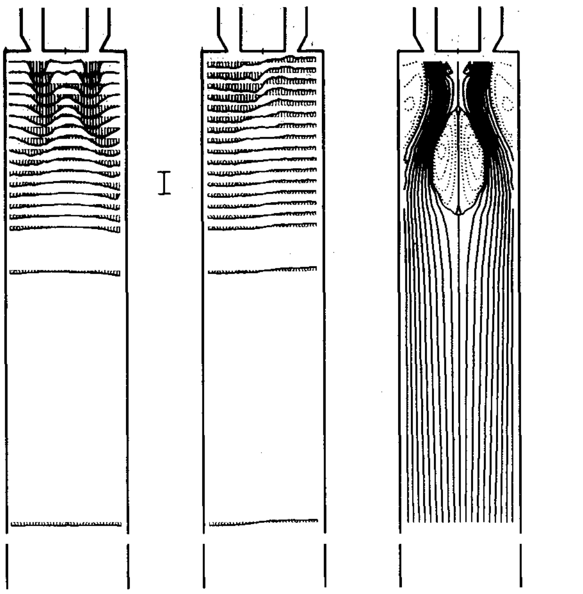

Fig. 4 LDA measurements for a flow which recovers to supercritical after breakdown: $\phi=62$ deg, $\beta=2.74$, $Re=7008$, with no exit contraction ($D_E/D=1$). a) Axial velocity, b) swirl velocity, c) streamline map. The vertical line between a and b represents a velocity of 1 m/s.

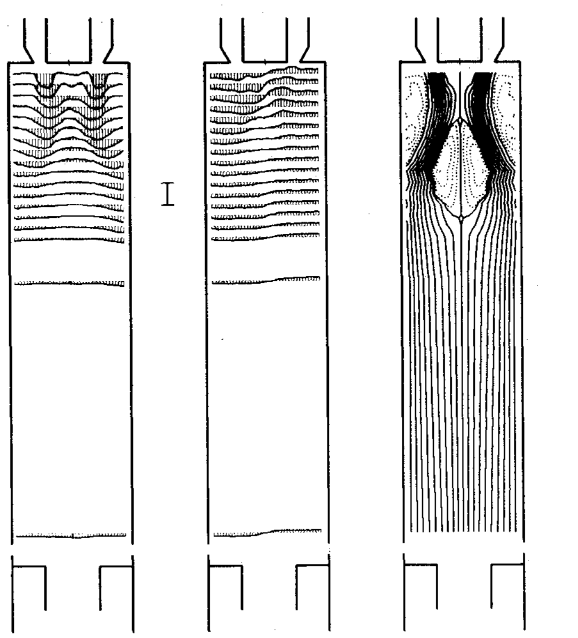

Fig. 5 LDA measurements for a flow which recovers to supercritical after breakdown with 79% exit-area contraction ($D_E/D=0.455$).

a Taylor expansion of Eq. (8) for small absolute values of $(x-x_c)$ yields to second order

$$y/y_c \cong 1 - \tfrac{1}{2}(1-R^2)(x-x_c)^2 \tag{10}$$

To understand the connection between the character of a recirculation zone and the downstream geometry of the vortex tube, a further aspect of jet- and wake-like cores should be discussed. Irrespective of the question whether a vortex core is produced by internal diffusion of vorticity (as in a Burgers vortex) or by convection of particles from the endwall boundary layer, the core radius decreases when the Reynolds number is increased. We consider a flow device with a fixed geometry consisting, for example, of a guidevane system (to produce a vortex with essentially irrotational flow outside the core) and a vortex tube with an area maximum, such as described earlier. The vortex flow will be subcritical

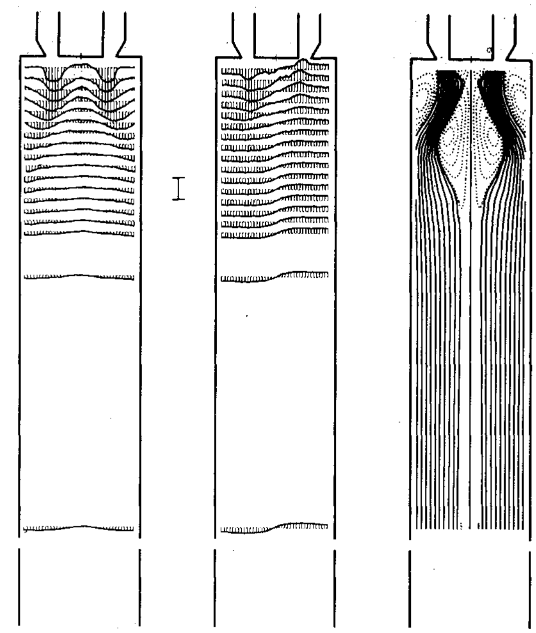

Fig. 6 LDA measurements for a flow which remains subcritical after breakdown: $\phi=70$ deg, $\beta=11.8$, $Re=7008$, with no exit contraction ($D_E/D=1$).

throughout the tube, provided the flow rate is chosen to be sufficiently large. When the flow rate is reduced, the vortex-core radius increases and, consequently, the flow condition is brought closer to critical everywhere in the tube. At a certain flow rate, the flow first becomes critical at the axial position which corresponds to the largest cross-sectional area. If the flow rate is now decreased further and neither vortex breakdown nor separation of the flow at the tube walls occurs, it must be the case that the flow remains critical at this location. However, as the core radius increases with decreasing flow rate, the flow at the area maximum can remain critical only if the velocity profile within the core changes appropriately. Thus, we see that the jet- or wake-like character of a vortex core is closely related to the appearance of a critical cross section in a vortex tube. Hence, the reason why the character of the axial velocity profile in a vortex core cannot be controlled if a critical cross section occurs is similar to the reason why the Mach number in a gas flow through a tube with a critical cross section cannot be controlled.

IV. Recirculation Zones

The contraction nozzle shown in Fig. 1 was incorporated for the LDA measurements presented in this section. The following dimensions of the flow device remained unchanged in all experiments: $D_L=20$ mm, $D_N=32$ mm, $D_0=40$ mm, $D=55$ mm, and $L=350$ mm. The flow rate in all cases $Q=0.29\ell/s$ with a corresponding Reynolds number $4Q/\pi D\nu=7000$ (for a mean water temperature of 22°C). For each of six cases measurements were made of the mean axial and swirl velocity components $u(r,x)$ and $v(r,x)$. The third diagram in each of Figs. 4-9 represents the streamline map calculated from the measured axial velocity. Minor asymmetries evident in the axial velocity distributions were averaged out prior to computing the normalized streamfunction ψ from

$$\psi = \int_0^r ur\mathrm{d}r \Big/ \int_0^{D/2} ur\mathrm{d}r$$

In the diagrams, the values of $\sqrt{\psi}$ on adjacent streamlines change linearly such that, for a uniform flow, the streamlines are equally spaced. The full lines correspond to $0 \le \psi \le 1$, and the broken lines to $\psi < 0$ or $\Psi > 1$ (i.e., recirculating flow).

For the first series of measurements (Figs. 4 and 5) the angle of attack of the swirl vanes $\phi = 62$ deg, chosen such that β ($=2.74$) exceeded β_0 only marginally. In this case viscous effects are sufficiently strong to allow the subcritical vortex flow to recover somewhere downstream of breakdown and again become supercritical upstream of the exit contraction. The supercritical character may be inferred from the results illustrated in Figs. 4 and 5 for exit diameters $D_E = 55$ mm (i.e., no contraction) and $D_E = 25$ mm. Although the exit contraction used in the case of Fig. 5 appears to represent a major change of the downstream geometry of the vortex tube, the flowfields shown in Figs. 4 and 5 are almost identical. There is a strong similarity between the streamline maps of Figs. 4 and 5 and that shown by Beer and Chigier.[13] Furthermore, a few diameters downstream of the inlet to the vortex tube, the axial velocity profile is quite uniform. Further evidence for the supercritical character of the downstream flow in Figs. 4 and 5 is obtained by considering the flow angle

$$\alpha(r) = \tan^{-1}[v(r)/u(r)] \tag{11}$$

for the velocity profiles at the furthest downstream location. Comparing the maximum flow angle $\alpha_M = \max(\alpha(r))$ with the maximum flow angle for a critical Rankine vortex α_c [see Eqs. (3), (5), (6), and (9)],

$$\alpha_c = \tan^{-1}(x_c/2) \tag{12}$$

and noting that

$$x_0 \leq x_c \leq x_1, \quad x_0 \cong 2.4, \quad x_1 \cong 3.8 \tag{13}$$

for all values of R, where x_0 is the smallest positive zero of J_0 and x_1 the smallest positive zero of J_1, it appears to be the case that

$$\alpha_M < \alpha_c \tag{14}$$

another indication that the downstream flow is supercritical. A further property of the flowfields illustrated in Figs. 4 and 5, which turns out to be essential, is the strictly wake-like character of the axial velocity profiles in the domain of the recirculation zone. This wake-like character is crucial for the stabilization of a flame in a swirl combustor.

For the flowfields illustrated in Figs. 6-9, the angle of the swirl vanes was $\phi = 70$ deg such that β ($=11.8$) clearly exceeded β_0 but was still somewhat smaller than β_1. In this case the flow remained subcritical in each case after the recirculation zone down to the end of the vortex tube. The only difference between these four cases is in the choice of the exit contraction (D_E). For Fig. 6, $D_E = 55$ (no contraction); for Fig. 7, $D_E = 47$; for Fig. 8, $D_E = 40$; and for Fig. 9, $D_E = 25$. The Reynolds number is again 7000. It is apparent that the exit contraction now has a strong influence on the entire flowfield. In the case of no contraction (Fig. 6), flow reversal is observed along the entire axis of the vortex tube; while for a strong contraction, the vortex core becomes strongly jet like. Also, the character of the recirculation zone changes quite significantly as the exit contraction is changed. For strong exit contractions, the recirculation zone assumes a mushroom

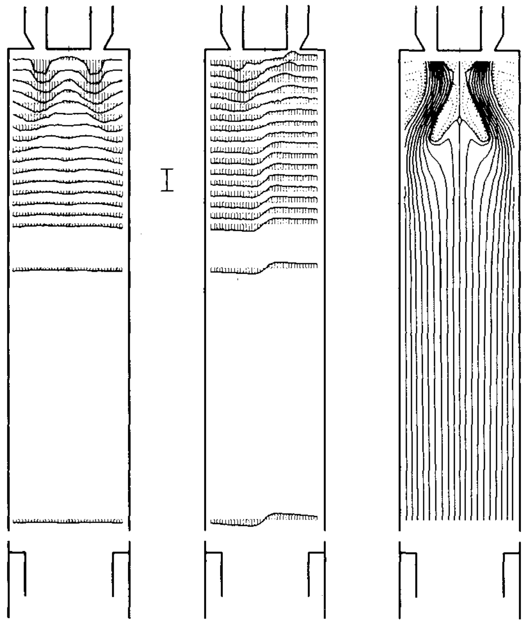

Fig. 8 LDA measurements for a flow which remains subcritical after breakdown with 47% exit-area contraction ($D_E/D = 0.727$).

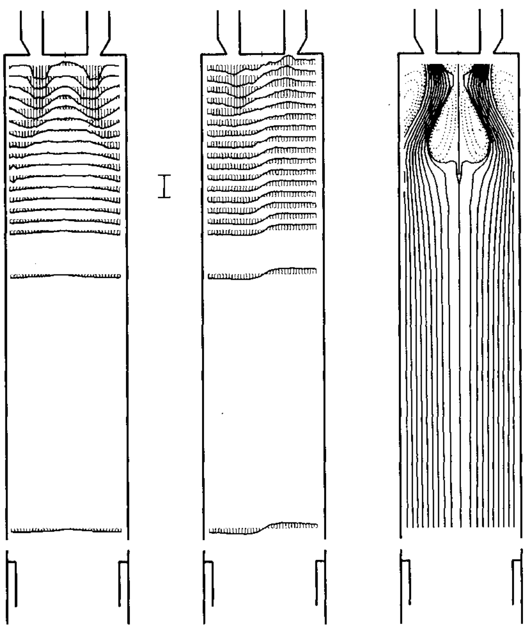

Fig. 7 LDA measurements for a flow which remains subcritical after breakdown with 27% exit-area contraction ($D_E/D = 0.855$).

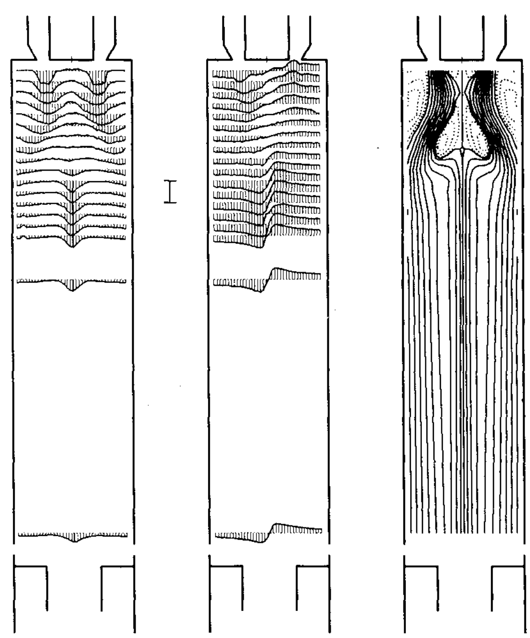

Fig. 9 LDA measurements for a flow which remains subcritical after breakdown with 79% exit-area contraction ($D_E/D = 0.455$).

shape, a character even more pronounced if the diameter of the upstream centerbody (gas lance) is reduced. For small centerbody diameters or in the absence of a centerbody, it is the case that the recirculation zone is limited to an annular lobe without flow reversal on the axis.[13] If there is no exit contraction or even a gradual increase of the vortex-tube diameter toward the exit, the corresponding flowfields are always rather similar to that shown in Fig. 7, provided the flow does not again become supercritical upstream of the exit due to other influences. If the flow does become supercritical again somewhere upstream of the end contraction, the contraction has no influence at all on the recirculation zone, as was the case for $\phi = 62$ deg. As no inertia waves can propagate upstream from the end contraction in this case, the recirculation zone does not "see" the exit of the vortex tube (compare Figs. 4 and 5).

In the light of the present results it is understandable that different investigators (with different experimental setups) have observed markedly different flowfields downstream of a vortex breakdown.

V. Concluding Remarks: Swirl Combustion

Typically, in the case of cold flow (no combustion) in a swirl combustor, a transition from supercritical to subcritical flow takes place across the recirculation zone and the vortex flow, in general, remains subcritical down to the exit of the combustion chamber. However, a significant difference must be allowed for in the presence of combustion. While the angular momentum of a particle changes little as the particle traverses the zone of reaction, the mean axial velocity is strongly increased due to the density reduction. As a result of this increase in axial velocity, the subcritical flow after vortex breakdown rapidly approaches the critical state and again becomes supercritical. In practical situations, the effect of combustion on the axial velocity is typically so strong that the vortex flow "recovers" to supercritical long before the end of the combustion zone. In other words, a vortex flow in a combustion chamber is, in general, only locally subcritical. The observations reported by Altgeld et al.,[8] for example, are entirely consistent with our interpretation of swirling flows. In the absence of combustion, backflow was evident throughout the combustor, and the entire flowfield strongly affected by the exit geometry. In the presence of combustion, in contrast, backflow did not occur. It is apparent that it is of fundamental importance for the combustion processes whether the flow is supercritical or subcritical downstream. Free and confined flames produced by the same swirl burner, for example, may have significantly different properties, because the reaction zone can be decoupled from the downstream flowfield by a suitable confinement of the vortex flow. It is also clear that extreme caution must be exercised in transferring the results of a nonreacting-flow investigation to the same flow and geometry with combustion. Also questionable is the practice of introducing modifications to "overcome" the undesirable consequences of subcritical flow, again exemplified by the investigation of Altgeld et al.[8] and, also, that of Kurosaka[19] in an entirely different application.

A further point which has been overlooked[20] until now concerns the problem of calculating the characteristics of swirling flows. It should be evident by this point that if the downstream flow is subcritical then great care must be taken over the specification of the downstream boundary conditions. In principle, the only completely acceptable approach would seem to be to continue the calculation to a location where the swirl has fallen to a sufficiently low level that the flow is again supercritical. A corollary is that the assumption that the flow in supercritical at some location may be inconsistent with the imposed upstream conditions.

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